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WAVE FLOW OF VISCOELASTIC SUSPENSIONS IN TUBES

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The wave propagation of viscoelastic suspensions in tubes is theoretically investigated, taking account of the deformation of the tube walls and the dispersity of the medium within the framework of the rheological model of a viscoelastic liquid with internal oscillators. A wave equation is obtained, and its limiting cases are analyzed. The dispersional relation is investigated with characteristic values of the rheological parameters of the medium. A numerical experiment is undertaken to investigate the influence of the rheology of the medium on the structure and dynamics of wave perturbation of velocity perturbations.

Introduction. The increased production of anomalous petroleum has prompted the active investigation of rheophysical problems of oil and gas production [1]. The high content of paraffin, naphthene, and aromatic hydrocarbons in the petroleum extracted and transported, which are present in the form of solid-phase disperse particles at the certain temperatures, means that the solid-hydrocarbon content may reach 18-20%. This leads to various anomalies in the rheodynamic properties of petroleum and hydrodynamic peculiarities in pipeline transport [2-3]. In particular, nonsteady wave conditions of flow appear in pipeline startup, with variation in pumping-station operating conditions, in emergency situations, etc. Experimental investigation of shock-wave propagation in paraffin petroleum and modeling of such media [4] shows the presence of new, previously undescribed features in the propagation of waves in petroleum. It is found that increase in solid-particle concentration leads to significant distortion of the structure and dynamics of shock-wave propagation. The distortion is such that it cannot be described within the framework of existing models of viscoelastic liquids [5]. In connection with this, there is a need to investigate the influence of rheological properties of suspensions on wave processes on the basis of fundamentally new models. The possibility of using the model of viscoelasticity with internal oscillators is considered below [6].

Since anomalous petroleum has viscoelastic properties [3], it is fairly difficult to determine the parameters of interphase interaction of such materials with disperse solid particles and hence to describe the media within the framework of a multispeed continuum. At the same time, taking into account that the densities of the liquid and solid phases are similar, and the particle dimensions are many times less than the distances between them, the tube diameters, and the given wavelengths, it is expedient to model the medium as quasi-homogeneous, neglecting the dynamic and inertial effects in the relative motion of the components. However, in this case, the medium is assumed to be continuous, and the presence of solid particles is only indirectly taken into account: by the change in rheological constants as a function of the concentration. This assumption of continuity of the medium eliminates the possibility of taking direct account of the influence of the particle dynamics on the wave propagation at a wavelength much greater than the particle size.

The problem of taking account of the dynamics of solid particles in a viscoelastic medium is analogous to that which arises in considering problems of nonlinear seismics [6]. On the basis of the analysis of experimental data on wave propagation in quartz and of various

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moisture contents, a rheological model of a medium with internal oscillators, in which the discreteness and influence of oscillations of the particles on the wave propagation is taken into account, was proposed in [6]. The model of [6] is chosen as the basis here in deriving and analyzing the equations of wave propagation in a viscoelastic suspension.

For the initial qualitative and quantitative analysis, it is expedient to use the simplest model of a linear viscoelastic liquid with internal oscillators relating to the tangential stress τ to the deformation rate $\dot{\gamma}$ in simple shear flow (an analog of the Maxwell model)

$$\tau + \lambda\dot{\tau} = \mu\dot{\gamma} + \lambda M\dddot{\gamma}. \tag{1}$$

A point above a symbol denotes total differentiation with respect to the time. The coefficient of the third derivative has the dimensions of mass per unit length, and may be interpreted as some linear density of the particles. Characterizing the medium as discrete, the coefficient M must contain information on the density of the material and the particle size and concentration. The numerical value of M, like the other rheological constants, may only be found experimentally for each specific medium.

Note that the model in Eq. (1) is only applicable in a limited frequency range: specifically, for $\omega < \omega_{cr} = (\mu/\lambda\mu)^{0.5}$. This constant follows from the condition that the scattering energy be positive after a cycle of oscillations with harmonic shear. For a paraffin-petroleum suspension, ω_{cr} is of the order of 10^2-10^3 sec^{-1}. Corresponding constraints are imposed on the shear rate, since $\dot{\gamma} \sim \omega$. Experimental investigations with a 20% suspension show that, up to shear rates of the order of 10^2 sec^{-1}, a linear dependence of the stress on the deformation rate is valid. Hence, in the given range of deformation-rate variation, the linear rheological model in Eq. (1) may be used.

A law for the tangential stress follows from Eq. (1) in the form

$$\tau = \frac{1}{\lambda}\int_{-\infty}^{t} (\mu\dot{\gamma} + \lambda M\dddot{\gamma}) \exp(-(t-t')/\lambda)\, dt'. \tag{2}$$

In considering the longitudinal waves, the dynamic equation in terms of the stress in a cylindrical coordinate system

$$\rho\frac{dV}{dt} = -\frac{\partial P}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) \tag{3}$$

is expediently averaged over the cross-sectional area of a tube of radius R_0. It is assumed here that the velocity gradient at the tube wall is proportional to the mean velocity U according to the hypothesis of quasi-steady conditions

$$\left(\frac{\partial V}{\partial r}\right)_{r=R_0} = (\dot{\gamma})_{r=R_0} = -\frac{4U}{R_0}.$$

The equation of motion in terms of the mean velocity of one-dimensional flow, taking account of Eq. (2), takes the form

$$\frac{dU}{dt} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} - \frac{8\mu}{\lambda R_0^2\rho_0} \times \int_{-\infty}^{t}\left(U + \frac{M\lambda}{\mu}\frac{\partial^2 U}{\partial t'^2}\right)\exp(-(t-t')/\lambda)\, dt'. \tag{4}$$

The complete system of equations also includes the mass-balance equation

$$\frac{\partial(\rho R^2)}{\partial t} + \frac{\partial(\rho U R^2)}{\partial x} = 0 \tag{5}$$

and the adiabatic equation of state under the assumption that there is no volume relaxation

$$\delta P = c_1^2\delta\rho. \tag{6}$$

The system of equations is closed by the relation between the pressure perturbation in the liquid and the change in radius of a liquid-filled tube, in the form

$$\delta P = \frac{Eh_T}{R_0^2}\delta P + \rho_T h_T\frac{\partial^2 R}{\partial t^2}. \tag{7}$$

No account is taken of the flexural rigidity of the tube.

Using the equations of simple waves as the first approximation, the system in Eqs. (4)-(7) reduces to two equations – Eqs. (4) and (8) – after eliminating ρ and R

$$\frac{\partial P}{\partial t}+U\frac{\partial P}{\partial x}+\rho_0 c^2\frac{\partial U}{\partial x}+\frac{\rho_T h_T R_0 c^4}{2c_2^4}\frac{\partial^3 U}{\partial t^2\partial x}=0. \tag{8}$$

Then, by cross integration of Eqs. (4) and (8) with respect to the time and the coordinate and subsequent subtraction of one from the other, a wave equation of hyperbolic type in terms of the velocity is obtained

$$\frac{\partial^2 U}{\partial t^2}-c^2\frac{\partial^2 U}{\partial x^2}-c\frac{\partial^2 U^2}{\partial x^2}-\frac{c^2}{\omega_T^2}\frac{\partial^4 U}{\partial x^2\partial t^2}+\frac{8\mu}{R_0^2\rho_0}\frac{\partial U}{\partial t}+$$
$$+\lambda\frac{\partial}{\partial t}\left(\mathrm{Pt}\frac{\partial^2 U}{\partial t^2}-c^2\frac{\partial^2 U}{\partial x^2}-c\frac{\partial^2 U^2}{\partial x^2}-\frac{c^2}{\omega_T^2}\frac{\partial^4 U}{\partial x^2\partial t^2}\right)=0. \tag{9}$$

In dimensionless variables, Eq. (9) takes a form expedient for analysis

$$\frac{\partial^2 \overline{U}}{\partial\theta^2}-\frac{\partial^2 \overline{U}}{\partial\xi^2}-\frac{\partial^2 \overline{U}^2}{\partial\xi^2}-\frac{\partial^4 \overline{U}}{\partial\xi^2\partial\theta^2}+\frac{8}{\mathrm{Rea}}\frac{\partial \overline{U}}{\partial\theta}+$$
$$+\mathrm{De}\frac{\partial}{\partial\theta}\left(\mathrm{Pt}\frac{\partial^2 \overline{U}}{\partial\theta^2}-\frac{\partial^2 \overline{U}}{\partial\xi^2}-\frac{\partial^2 U^2}{\partial\xi^2}-\frac{\partial^4 \overline{U}}{\partial\xi^2\partial\theta^2}\right)=0. \tag{10}$$

Here Pt is a parameter characterizing the influence of the oscillating masses; the acoustic Reynolds number Rea determines the relative contribution of viscous forces, and the Debord number De determines the relation between the relaxation time and the relative time of the process T.

On the basis of Eq. (10), comprehensive analysis of the evolution of one-dimensional velocity perturbations is possible. The influence of both the tube parameters and the rheological properties of the suspension are taken into account here. The radius, wall thickness, density, and elastic modulus of the tube material determine its inertial properties, the influence of which on the structure and dynamics of propagation of the perturbations is found to reduce to the appearance of dispersional effects [7]. Hence, attention must focus basically on the influence of the rheophysical characteristics of the suspension.

Linear Waves. If the amplitude of the perturbations is small, the nonlinear terms in Eq. (10) may be neglected; then

$$\frac{\partial^2 \overline{U}}{\partial\theta^2}-\frac{\partial^2 \overline{U}}{\partial\xi^2}-\frac{\partial^4 \overline{U}}{\partial\xi^2\partial\theta^2}+\frac{8}{\mathrm{Rea}}\frac{\partial \overline{U}}{\partial\theta}+\mathrm{De}\frac{\partial}{\partial\theta}\left(\mathrm{Pt}\frac{\partial^2 \overline{U}}{\partial\theta^2}-\frac{\partial^2 \overline{U}}{\partial\xi^2}-\frac{\partial^4 \overline{U}}{\partial\xi^2\partial\theta^2}\right)=0. \tag{11}$$

From the viewpoint of the rheology of viscoelastic media, the influence of the relaxation time on the wave evolution is of most interest. The following limiting cases are considered.

1. The relaxation time is much less than the characteristic time, i.e., De ≪ 1. In this case, Eq. (11) takes the form

$$\frac{\partial^2 \overline{U}}{\partial\theta^2}-\frac{\partial^2 \overline{U}}{\partial\xi^2}-\frac{\partial^4 \overline{U}}{\partial\xi^2\partial\theta^2}+\frac{8}{\mathrm{Rea}}\frac{\partial \overline{U}}{\partial\theta}=0. \tag{12}$$

Equation (12) corresponds to a wave propagating in a medium with dispersion due to the deformation of the tube walls and dissipation due to the viscosity of the suspension. The relaxational properties of the medium in this case have no influence on the wave evolution.

2. The relaxation time is much greater than the wave period, i.e., D ≫ 1. In this case, Eq. (11) takes the form (under the assumption that dissipative effects are significant)

$$\mathrm{Pt}\frac{\partial^2 \overline{U}}{\partial\theta^2}-\frac{\partial^2 \overline{U}}{\partial\xi^2}-\frac{\partial^4 \overline{U}}{\partial\xi^2\partial\theta^2}-\frac{8}{\mathrm{De\,Rea}}\overline{U}=0. \tag{13}$$

Equation (13) corresponds to a wave propagation in a medium with dispersion and dissipation; the relaxational properties of the medium and the presence of solid particles have a significant influence. With increase in relaxation time, corresponding to increase in

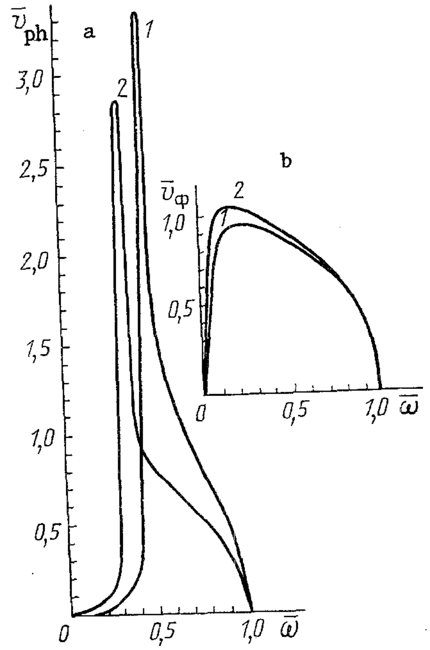

Fig. 1. Distribution of reduced phase velocity $\bar{V}_{ph}$ with respect to frequency $\bar{\omega}$ at various values of rheological parameters.

De, there is effective decrease in the dissipation. This is analogous to the effect described for viscoelastic relaxing liquids with gas bubbles [8]. It is also evident from Eq. (13) that the velocity of wave propagation decreases with increase in solid-particle content and corresponding increase in Pt.

If the solution of Eq. (13) is sought in the form of a traveling plane wave $\bar{U}=\exp(i(\bar{\omega}\theta-\bar{k}\xi))$, the relation between the wave vector $\bar{k}$ and the frequency $\bar{\omega}$ takes the form

$$\bar{k}^2=\frac{\mathrm{Pt}\bar{\omega}^2-8/(\mathrm{De\,Rea})}{1-\bar{\omega}^2}. \quad (14)$$

The frequency dependence of the phase velocity $\bar{V}_{ph}$ may be determined from Eq. (14)

$$\bar{V}_{ph}=\left(\frac{1-\bar{\omega}^2}{1-8/(\mathrm{De\,Rea}\,\bar{\omega}^2)}\right)^{0.5}. \quad (15)$$

Curves of the phase velocity plotted from Eq. (15) are shown in Fig. 1a, where Rea = 0.8, De = 50.0, Pt = 1.0 (1) and 2.0 (2). The fundamental difference between these curves and those in Fig. 1b is their form, indicating that in the spectrum of the wave there is a lower bound on the frequency $\bar{w}=(8/(\mathrm{DeReaPt}))^{0.5}$ and an upper bound $\bar{w}=1$. A "window of transparency" is formed, by analogy with the well-known window of nontransparency of the medium [9]. It is noteworthy that shift in the phase-velocity maximum to lower frequency is observed with increase in Pt (increase in solid-particle content). Reduction in the maximum phase velocity occurs here.

The curves in Fig. 1b are plotted from the dispersion relation corresponding to Eq. (11)

$$\bar{V}_{ph}^2=\frac{(1-\bar{\omega}^2)(1-i\,\mathrm{De}\,\bar{\omega})(1-i(8/(\mathrm{Rea}\,\bar{\omega})-\bar{\omega}\,\mathrm{De\,Pt}))}{1+(8/(\mathrm{De}\,\bar{\omega})-\bar{\omega}\,\mathrm{De\,Pt})^2} \quad (16)$$

with the following parameter values: 1) Rea = 15.0; De = 0.01; Pt = 1.0 (the case when De ≪ 1); 2) Rea = 15.0; De = 5.0; Pt = 1.0 (the case when De ~ 1). It is evident that with increase in De, i.e., with increase in relaxation time, there is an increase in phase velocity, corresponding to effective decrease in dissipation.

Curves of the phase velocity in Eq. (16) are shown in Fig. 2 with Rea = $8\cdot10^4$ (negligible dissipation), De = 100.0, and Pt = 1.0 (1), 1.5 (2), and 2.0 (3). It is evident that, with increase in Pt, the phase velocity decreases over the whole frequency spectrum. This confirms the conclusion that there is decrease in the velocity of wave propagation with increase in solid-particle content.

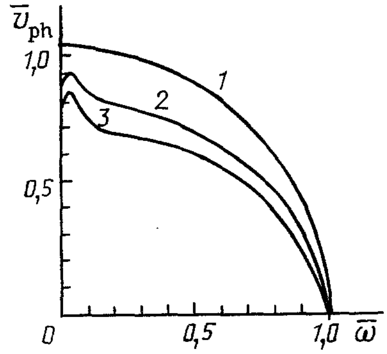

Fig. 2

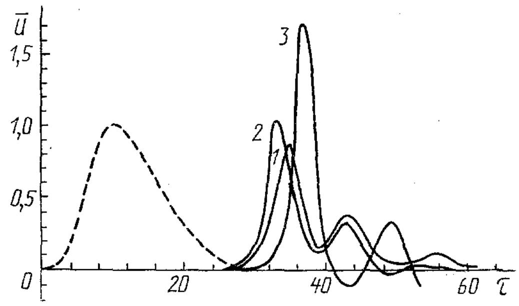

Fig. 3

Fig. 2. Decrease in phase velocity $\bar{V}_{ph}$ with increase in content of oscillating masses.

Fig. 3. Time dependence of velocity perturbation $\bar{U}$ at various values of De.

Numerical Solution of Nonlinear Equation. To elucidate the role of the rheological parameters of the medium in the structure and dynamics of the propagation of nonlinear perturbations, numerical integration of Eq. (10) is undertaken with zero initial condition and a condition at the boundary taken in the form of a Gaussian distribution

$$\begin{aligned} \bar{U}(0,\ \theta) &= \exp(-(3\mathrm{fr} - \theta/\mathrm{ti})^2),\ \theta < 3\mathrm{fr\,ti}, \\ \bar{U}(0,\ \theta) &= \exp(-(3 - \theta/\mathrm{ti\,fr})^2),\ \theta \geqslant 3\mathrm{fr\,ti}. \end{aligned} \tag{17}$$

The condition in Eq. (17) corresponds to the appearance of a finite velocity perturbation at the left-hand end of the tube; the form and length of the perturbation are determined by fr and ti.

The equation is approximated by an implicit five-layer finite-difference scheme of second order in ξ and first order in θ. The integration steps are chosen from the requirement of stability of the difference scheme. As a result of integration, the time dependence of the velocity in the specified tube cross section ξ^* is constructed.

Different values of De are taken in the integration of Eq. (10). The results of the numerical experiment confirm the conclusion that there is effective decrease in dissipation with increase in relaxation time. The behavior of the perturbations in the cross section $\xi^* = 30.0$ is shown in Fig. 3. The dashed curve shows the velocity perturbation at the left-hand end of the tube with $\xi = 0.0$ (ti = 8.0; fr = 0.5). Curve 1 corresponds to Rea = 15.0, De = 0.5, Pt = 1.0, and curve 2 to Rea = 15.0, De = 5.0, Pt = 1.0. It is evident that the amplitude of the perturbation waves increases with increase in De. At large relaxation times, the influence of the solid particles becomes significant. Curve 3 corresponds to Rea = 15.0, De = 5.0, Pt = 1.5. Here increase in amplitude and simultaneous decrease in velocity of propagation of the wave is noted with increase in Pt.

Slowing of the perturbation-wave propagation and simultaneous increase in its amplitude with increase in Pt is also seen in Fig. 4, where Rea = 8.0, De = 50.0, Pt = 1.0 (1), 1.5 (2), and $\xi^* = 1.5$, i.e., the combination of parameters corresponds to the limiting case

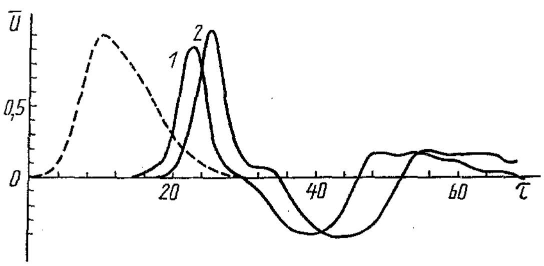

Fig. 4. Time dependence of nonlinear velocity perturbation $\bar{U}$ corresponding to a linear wave with a "window of transparency."

in Eq. (13). It is evident that the structure of the wave differs considerably from that in Fig. 3. The appearance of a large rarefaction region moving behind the leading front of the wave is characteristic here.

Conclusions. Analysis of perturbation-wave propagation in a viscoelastic suspension on the basis of a mechanical model with oscillating masses permits the following conclusion. Increase in the viscoelastic characteristics of the suspension, which may be due to increase in the content of paraffin fractions in the petroleum or reduction in temperature, leads to increase in relaxation time. This leads to increase in perturbation amplitude in wave propagation in pipelines and must be taken into account in hydrodynamic calculations of the technological operating conditions of pipelines and other oil-industry equipment. At the same time, increase in solid-particle concentration also leads to increase in perturbation amplitude and decrease in propagation velocity. These data are in complete agreement with the experimental results obtained in a shock tube with shock-wave propagation in petroleum suspensions.

Notation. c, r, cylindrical coordinates; t, time; V, longitudinal velocity coordinate; P, pressure; ρ, density of suspension; τ, $\dot{\gamma}$, tangential stress and deformation rate in simple shear flow; λ, μ, relaxation time and viscosity of suspension; M, density of oscillating masses; U, mean longitudinal velocity; R, tube radius; E, ρ_T, elastic modulus and density of tube material; h_T, thickness of tube wall; c_1, velocity of sound in the suspension; $c_2 = \sqrt{Eh_T/2R_0\rho_0}$, velocity of sound in the wall of a tube (radius R_0) containing liquid of density ρ_0; $c = c_1c_2/\sqrt{c_1^2 + c_2^2}$, Korteweg–Zhukovskii sound velocity; $\omega_T = c_2^2\sqrt{2\rho_0}/c\sqrt{\rho_T h_T R_0}$, oscillation frequency of tube wall; $T = 1/\omega_T$, characteristic time of process; $De = \lambda/T$, Debord number; $Rea = R_0^2\rho_0\omega_T/\mu$, acoustic Reynolds number; $Pt = 1 + 8M/R_0^2\rho_0$, parameter characterizing the influence of the oscillating masses; P_0, ρ_0, R_0, unperturbed values of the pressure, the density of the suspension, and the tube radius. Dimensionless variables: $\bar{U}$, velocity; $\theta = t\omega_T$, time; $\xi = x\omega_T/c$, longitudinal coordinate; $\bar{\omega} = \omega/\omega_T$, frequency; $\bar{k} = kc/\omega_T$, wave number; $\bar{V}_{ph} = V_{ph}/c$, phase velocity; fr, ti, parameters determining the length and curvature of the leading wavefront.

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